FYBSc IT | Discrete Maths | 100

Q.P. Code : 750401

(2½ Hours)

[Total Marks: 75

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. **Attempt *any three* of the following:** **15**

a. Define the following:
   i. Universal statement
   ii. Existential universal statement
   iii. Subset
   iv. Cartesian product
   v. Relation

b. i. Which of the following sets are equal? Justify your answer.
   $A = \{0, 1, 2\}$
   $B = \{x \in R \mid -1 \leq x \leq 3$
   $C = \{x \in R \mid -1 < x < 3$
   $D = \{x \in Z \mid -1 < x < 3$
   $E = \{x \in Z^+ \mid -1 < x < 3$
   ii. For each integer n, $T_n = \{n, n^2\}$. How many elements are there in each of T_2, T_{-3}, T_1, T_0. Justify your answers.

c. For each positive integer i, $A_i = \left\{x \in R \mid -\frac{1}{i} < x < \frac{1}{i}\right\} = A_i = \left(-\frac{1}{i}, \frac{1}{i}\right)$
   i. Find $A_1 \cup A_2 \cup A_3$ and $A_1 \cap A_2 \cap A_3$
   ii. Find:
   $$\bigcup_{i=1}^{\infty} A_i \text{ and } \bigcap_{i=1}^{\infty} A_i$$

d. Explain Russell's Paradox with an example.

e. Write the following statements using the symbols ~, Λ, V and the indicated letters to represent the component statements. h: "Raj is healthy", w: "Raj is wealthy", s: "Raj is wise"
   i. Raj is healthy and wealthy but not wise.
   ii. Raj is not wealthy but he is healthy and wise.
   iii. Raj is neither healthy, wealthy nor wise.
   iv. Raj is neither wealthy, nor wise but he is healthy.
   v. Raj is wealthy but he is not both healthy and wise.

f. In the back of an old cupboard there is a note signed by a pirate famous for his bizarre sense of humor and love of logical puzzles. In the note he wrote that he had hidden treasure somewhere on the property. He listed five true statements listed below and challenged the reader to use them to figure out the location of the treasure. Write the sequence of steps and locate the treasure.
   i. If this house is next to a lake, then the treasure is not in the kitchen.
   ii. If the tree in the front yard is an elm, then the treasure is in the kitchen.
   iii. This house is next to a lake.
   iv. The tree in the front yard is an elm or the treasure is buried under the flagpole.
   v. If the tree in the back yard is an oak, then the treasure is in the garage.
   Where is the treasure?

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2. Attempt *any three* of the following: 15

a. Define a predicate and its truth set. Let $P(x)$ be a predicate "$x^2 > x$" with domain the set **R** of all real numbers. Write P(2), P(½) and P(−½) and indicate, which of these statements are true and which are false.

b. i. Using the laws for negating universal and existential statements, derive the following rules:

$\sim(\forall x \in D(\forall y \in E(P(x, y)))) \equiv \exists x \in D(\exists y \in E(\sim P(x, y)))$ and

$\sim(\exists x \in D(\exists y \in E(P(x, y)))) \equiv \forall x \in D(\forall y \in E(\sim P(x, y)))$

ii. Indicate which of the following statements are true and which are false. Justify your answers.

- $\forall x \in Z^+, \exists y \in Z^+$ such that $x = y + 1$.
- $\forall x \in Z^+$ and $\forall y \in Z^+, \exists z \in Z^+$ such that $z = x - y$.

c. Indicate whether the following arguments are valid or invalid. Support your answer with diagrams:

i. All human beings are mortal.
Raju is mortal.
∴ Raju is human being.

ii. All polynomial functions are differentiable.
All differentiable functions are continuous.
∴ All polynomial functions are continuous.

d. Define prime numbers and composite numbers. Express the definition using symbols. Prove that every integer greater than 1 is either prime or composite. Write first six prime numbers and composite numbers.

e. i. State the quotient remainder theorem.
ii. Today is Friday (11/11/2016). 2017 is not a leap year. Find the day of week, 1 year from today.
iii. Suppose m is an integer. If m mod 11 = 6, what is $4m$ mod 11?

f. State the Euclidian algorithm. Find the gcd of (330,156) by using Euclidean algorithm,

3. Attempt *any three* of the following: 15

a. Using the method of induction prove that:

$\left(1-\frac{1}{2^2}\right)\cdot\left(1-\frac{1}{3^2}\right)\cdots\left(1-\frac{1}{n^2}\right) = \frac{n+1}{2n}\ for\ all\ integers\ n \geq 2.$

b. Prove that if the given predicate is true before entering the loop, it is true after exiting the loop.

loop: while ($n \geq 3$ *and* $n \leq 100$)
$n := n + 1$
end while

predicate: $2n + 1 \leq 2^n$

c. Suppose a sequence $b_0, b_1, b_2, \ldots$ satisfies the recurrence relation $b_k = 4b_{k-1} + 4b_{k-2}$ for all integers $b \geq 2$ with initial conditions b_0=1 and b_1 = 3. Find the explicit formula for $b_0, b_1, b_2, \ldots$

d. Define (i) Function (ii)Logarithm (iii) Logarithmic function (iv) Boolean function (v) Image and Inverse Image.

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e. Define surjective function and inverse function. Find the inverse of the following functions:

  i. Define $f: \mathbf{Z} \to \mathbf{Z}$ by the rule $f(n) = 2n$ for all integers n.
  ii. Define G: $\mathbf{R} \to \mathbf{R}$ by the rule $G(x) = 4x - 5$ for all real numbers x.

✓f. Let $f: \mathbf{Z} \to \mathbf{Z}$ be the successor function and let $g: \mathbf{Z} \to \mathbf{Z}$ be the squaring function. Then $f(n) = n + 1$ for all $n \in \mathbf{Z}$ and $g(n) = n^2$ for all $n \in \mathbf{Z}$.

  i. Find the compositions $g \circ f$ and $f \circ g$.
  ii. b. Is $g \circ f = f \circ g$? Explain.

4. Attempt *any three* of the following:

a A Relation R from $\mathbf{R}$ to $\mathbf{R}$ is defined as follows: For all $(x, y) \in \mathbf{R} \times \mathbf{R}$.

$$x\,R\,y \Leftrightarrow y = 2|x|.$$

Draw the graphs of R and R^{-1} in the Cartesian plane. Is R^{-1} a function?

b The congruence modulo 3 relation T on $\mathbf{Z}$ is defined as follows: For all integers m and n,

$$m\,T\,n \Leftrightarrow 3 \mid (m - n).$$

i. Is T reflexive? ii. Is T symmetric? iii. Is T transitive?

c A relation R on A is defined as follows:

For all $(a, b), (c, d) \in A$, $(a, b)\,R\,(c, d) \Leftrightarrow ad = bc$.

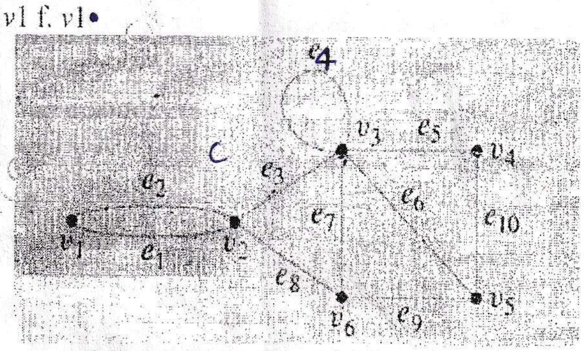
  i. Prove that R is transitive.
  ii. Describe the distinct equivalence classes of R.

✓d Define graph, digraph, simple graph and subgraph. Draw a graph with the specified properties or show that no such graph exists.

  i. A graph with four vertices of degrees 1, 1, 2, and 3
  ii. A graph with four vertices of degrees 1, 1, 3, and 3
  iii. A simple graph with four vertices of degrees 1, 1, 3, and 3

✓e In the graph below, determine which of the following walks are trails, paths, circuits, or simple circuits. Justify your answer.

  i. v1e1v2e3v3e4v3e5v4
  ii. e1e3e5e5e6
  iii. v2v3v4v5v3v6v2
  iv. v2v3v4v5v6v2 C
  v. v1e1v2e1v1 f. v1•



f Explain rooted tress and binary trees. Draw binary trees to represent the following expressions:

  i. $((a - b) \cdot c) + (d/e)$
  ii. $a \cdot b - (c / (d + e))$

5. **Attempt *any three* of the following:** 15

a. One urn contains one blue ball (labeled $B1$) and three red balls (labeled $R1$, $R2$, and $R3$). A second urn contains two red balls ($R4$ and $R5$) and two blue balls ($B2$ and $B3$). An experiment is performed in which one of the two urns is chosen at random and then two balls are randomly chosen from it, one after the other without replacement.

   i. Construct the possibility tree showing all possible outcomes of this experiment.

   ii. What is the total number of outcomes of this experiment?

   iii. What is the probability that two red balls are chosen?

b. i. In a group of six people, must there be at least two who were born in the same month? In a group of thirteen people, must there be at least two who were born in the same month? Why?

   ii. A drawer contains ten black and ten white socks. You reach in and pull some out without looking at them. What is the least number of socks you must pull out to be sure to get a matched pair? Explain how the answer follows from the pigeonhole principle.

c. If n is a positive integer, how many 4-tuples of integers from 1 through n can be formed in which the elements of the 4-tuple are written in increasing order but are not necessarily distinct?

d. Use Pascal's formula to prove by mathematical induction that if n is an integer and $n \geq 1$, then

$$\sum_{i=2}^{n}\binom{i}{2}=\binom{2}{2}+\binom{3}{2}+\cdots+\binom{n+1}{2}=\binom{n+2}{3}$$

e. Suppose a person offers to play a game with you. In this game, when you draw a card from a standard 52-card deck, if the card is a face card you win Rs. 3, and if the card is anything else you lose Re. 1. If you agree to play the game, what is your expected gain or loss?

f. Consider a medical test that screens for a disease found in 5 people in 1,000. Suppose that the false positive rate is 3% and the false negative rate is 1%. Then 99% of the time a person who has the condition tests positive for it, and 97% of the time a person who does not have the condition tests negative for it.

   i. What is the probability that a randomly chosen person who tests positive for the disease actually has the disease?

   ii. What is the probability that a randomly chosen person who tests negative for the disease does not indeed have the disease?