

- Note :**
- 1) All Questions are compulsory.
 - 2) Make suitable assumptions wherever necessary.
 - 3) Figures to right indicate full marks.
 - 4) Use of Non-Programmable calculator is allowed.

Q.1. Attempt any three of following.

15 M

- a. If $A = \{2, 4, 6\}$ and $B = \{1, 3, 5\}$, Find $A \times B$, $A \times A$, $B \times B$, $B \times A$
- b. Let D be the set of positive divisors of 15.
Define : $a + b = \text{L.C.M. } \{a, b\}$ for all $a, b \in D$
 $a \cdot b = \text{GCD } \{a, b\}$ for all $a, b \in D$
show that $(D, +, \cdot, ', 1, 15)$ is Boolean Algebra.
- c. Obtain the truth table for
 - i. $\sim p \rightarrow q$
 - ii. $(p \vee q) \rightarrow r$
 - iii. $(p \rightarrow r) \wedge (q \rightarrow r)$
- d. Identify whether the following statements are valid or invalid.
 - i. This real number is rational or it its irrational
This real number is not rational
 \therefore Real number is irrational
 - ii. Murder is always wrong
Sometimes murder isn't wrong
 \therefore Death penalty should be illegal
- e. Rewrite the following statements informally in at least two different ways without using variables or quantifiers.
 - i. \forall rectangles x , x is a quadrilateral
 - ii. \exists a set A , such tat A has 16 subsets.
- f. State & prove De-Morgan's Law.

Q.2. Attempt any three of following.

15 M

- a. Prove that for all integers n , $n^2 - n + 11$ is a prime number.
- b. Prove that two consecutive integers have opposite parity.
- c. Prove that $\sqrt{2}$ is an irrational number.
- d. Let a, b, c be integers, such that a/b an a/c then show that $a | (b+c)$
- e. Prove that $\lfloor x + 3 \rfloor = \lfloor x \rfloor + 3$
- f. The sum of any two even integers is even.

Q.3. Attempt any three of following.

15 M

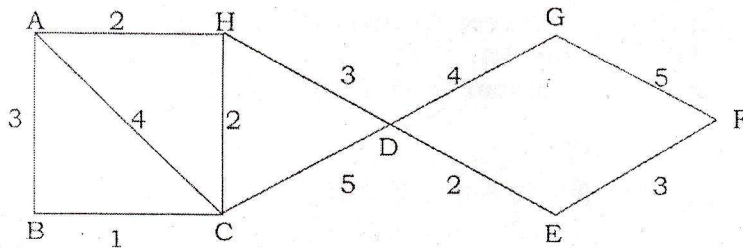
- a. Using mathematical induction prove that for all integers $n \geq 0$, $2^{2n} - 1$ is divisible by 3.
- b. Solve the following recurrence relation
 $t_n = 5t_{n-1} - 6t_{n-2}$ subjects to initial condition $t_0 = 7, t_1 = 16$
- c. Define followings.
 1. One-one function
 2. Onto function
 3. Bijective function
 4. Inverse function
- d. Given $f(x) = 3x+2$ and $g(x) = x+5$ find $f \circ g$ & $g \circ f$, is $f \circ g = g \circ f$
- e. Determine whether given function is bijective
 $f(x) = (x+1) / (x+2)$
- f. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = 6x+4$, find f^{-1} if it exists.

Q.4. Attempt any three of the following.

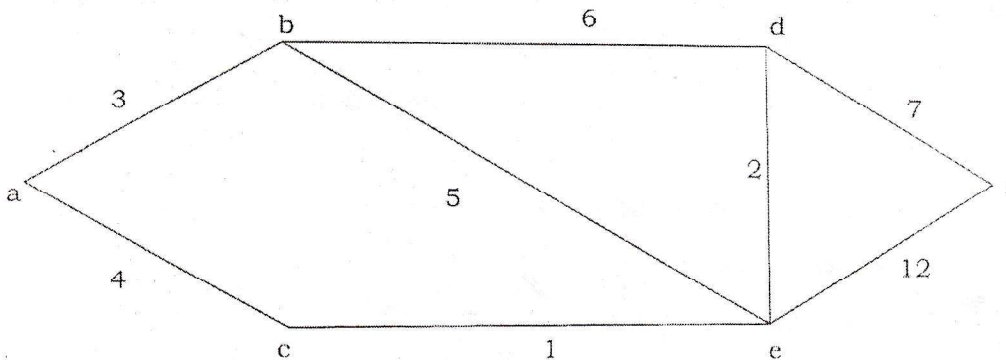
15 M

- a. Define following terms.
 1. Loops
 2. Degree of vertex
 3. Walk
 4. trail

- b. Using Kruskal's algorithm find minimum spanning tree.



- c. Let R be the relation defined on Set $A = \{1, 2, 3, 4\}$ as
 $R = \{(1, 1), (1,2), (3,1), (2,2), (3,2), (4,3), (4,4)\}$
 Find i) R^{-1} ii) MR (Matrix Representation of Rel^n) iii) Diagram of R
- d. Give example of a relation on set $A = \{a, b, c, d\}$ which satisfies
 i) Reflexive, Symmetric but not transitive
 ii) Reflexive, transitive but not symmetric
 iii) Equivalence Relation
- e. If A & B two events such that $P(A) = 0.8$; $P(B) = 0.7$, $P(A \cup B) = 0.6$
 Find $P(A \cap B)$, $P(A/B)$, $P(B/A)$
- f. Find the shortest path using Dijkstra's algorithm.



Q.5. Attempt any three of following.

15 M

- How many license plates can be made using either three digits followed by upper case English letter or three upper case English letters followed by three digits?
- Show that whenever 25 Girls & 25 Boys are seated around circular table there is always person both of whose neighbours are boys.
- Suppose the group of 12 consist of 5 men and 7 women.
 - How many 5-person team can be chosen that consist of 3 Men & 2 Women
 - How many 5-person team contain at most one man?
- Two unbiased coins are tossed at a time. Find the probability of obtaining
 - Exactly one head
 - No Head
 - At least one head
- If A & B are independent events such that
 $P(A) = 3/10$, $P(B) = 3/5$ find $P(A')$, $P(B')$, $P(A \cap B)$
 $P(A' \cap B)$, $P(A' \cap B')$, $P(A \cup B)$ and $P(A' \cup B)$
- An urn consist of 3 red & 4 green balls. Find expected value of red ball drawn.