

Q.P. Code : 781300

(2½ Hours)

[Total Marks: 75]

- N.B. (1) All questions are compulsory.  
(2) Figures to the right indicate marks.

## 1. Answer the following questions

(15 M)

## (a) Choose the best choice for the following questions:

(5 M)

- (i) If  $f_1$  and  $f_2$  are two functions from  $\mathbf{R}$  to  $\mathbf{R}$  such that  $f_1(x) = x^2$  and  $f_2(x) = x - x^2$ , then  $(f_1 \cdot f_2)(x)$  is given by  
(a)  $x^3 + x^4$  (b)  $x^3 - x^4$   
(c)  $x^4 + x^3$  (d) None of these
- (ii) Which of the following is true about the poset  $(\mathbf{Z}^+, |)$ ?  
(a) Zero is the least element (b) One is the least element  
(c) There is no least element (d) None of these
- (iii) A class contains 10 students with 6 men and 4 women. Number of ways to select a 4-member committee with 2 men and 2 women is given by  
(a) 60 (b) 70 (c) 80 (d) 90
- (iv) Suppose a bookcase shelf has 5 History texts, 3 Sociology texts, 6 Anthropology texts, and 4 Psychology texts. Number of ways a student can choose one text of each type is given by  
(a) 360 (b) 460 (c) 560 (d) 660
- (v) A loop is an edge connecting  
(a) a vertex with itself (b) two distinct vertices  
(c) no vertices (d) three distinct vertices

## (b) Fill in the blanks for the following questions:

(5M)

- (i) A function  $f$  is said to be strictly \_\_\_\_\_ if  $f(x) > f(y)$  for any  $x$  and  $y$  in the domain of  $f$ .
- (ii) A relation  $R$  on a set  $A$  is called \_\_\_\_\_ if  $(a, a) \in R$  for every element  $a \in A$ .
- (iii) The Gödel number of a word  $w = a_3a_2a_1a_3a_4$  is \_\_\_\_\_.
- (iv) Suppose that a procedure can be broken down into two tasks. If there are  $n_1$  ways to do the first task and  $n_2$  ways to do the second task after the first task has been done, then there are \_\_\_\_\_ ways to do the procedure.
- (v) Let  $G$  be a directed graph and  $v$  be a vertex of  $G$ . The number of edges beginning at  $v$  is called \_\_\_\_\_.

## (c) Answer the following questions:

(5M)

- (i) Why is  $f$ , defined by  $f(x) = 1/(x+1)$ , not a function from  $\mathbf{R}$  to  $\mathbf{R}$ ?
- (ii) Let  $\{a_n\}$  be a sequence that satisfies the recurrence relation  $a_n = a_{n-1} + 2$  for  $n = 1, 2, 3, \dots$  and suppose that  $a_0 = 2$ . What are  $a_1$  and  $a_2$ ?

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- (iii) State inclusion exclusion principle.
- (iv) Define a regular grammar.
- (v) Define a directed graph.

2. Answer any *three* of the following:

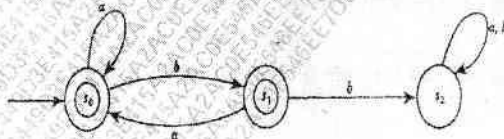
(15 M)

- (a) Find the domain and range of the following functions:
  - i) the function that assigns to each nonnegative integer its last digit.
  - ii) the function that assigns to a bit string the number of bits in the string.
- (b) Determine whether the function  $f$  from  $\mathbb{R}$  to  $\mathbb{R}$  defined by  $f(x) = x - 2$  is bijective.
- (c) Suppose that  $R$  is the relation on the set of strings of English letters such that  $aRb$  if and only if  $l(a) = l(b)$ , where  $l(x)$  is the length of the string  $x$ . Is  $R$  an equivalence relation? Justify your answer.
- (d) Define Lattice. Determine whether the posets  $(\{1, 2, 3, 4, 5\}, |)$  and  $(\{1, 2, 4, 8, 16\}, |)$  are lattices.
- (e) Solve the recurrence relation  $a_n = 5a_{n-1} - 6a_{n-2}$  for  $n \geq 2$ ,  $a_0 = 1$ ,  $a_1 = 0$ .
- (f) Define Fibonacci numbers. Formulate a recurrence relation for Fibonacci numbers.

3. Answer any *three* of the following:

(15 M)

- (a) How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other?
- (b) State and prove Vandermonde's identity.
- (c) State Pigeonhole principle. From any set of 13 integers, prove that there will be at least one pair which leaves the same remainder when divisible by 12.
- (d) How many integers between 1 and 600 (both inclusive) are not divisible by both 3 and 5?
- (e) Define a language  $L$  over an alphabet  $A$ . Let  $A = \{a, b, c\}$ . Find  $L^*$  where language  $L = \{b^2\}$ .
- (f) Determine whether or not the automaton  $M$  in the following figure accepts the words:  $w_1 = ababba$ ;  $w_2 = baab$ ;  $w_3 = \lambda$  the empty word.

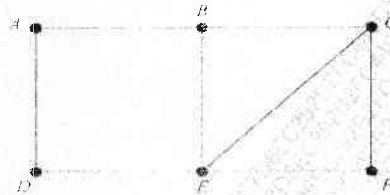


4. Answer any *three* of the following:

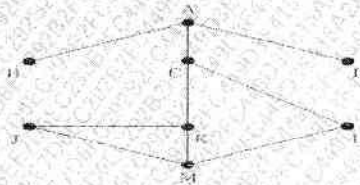
(15 M)

- (a) Consider the graph  $G$  in the following. Find (i)  $\text{diam}(G)$ , the diameter of  $G$ , (ii)  $d(A, F)$ , the distance from  $A$  to  $F$ .

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- (b) Consider the graph  $G$  in the following figure (where the vertices are ordered alphabetically). (i) Find the adjacency structure of  $G$ . (ii) Find the order in which the vertices of  $G$  are processed using a depth-first search algorithm beginning at vertex  $A$ .



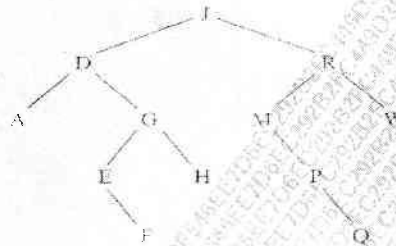
- (c) Draw the graph  $G$  corresponding to each adjacency matrix:

$$\begin{bmatrix} 1 & 3 & 0 & 0 \\ 3 & 0 & 1 & 1 \\ 0 & 1 & 2 & 2 \\ 0 & 1 & 2 & 0 \end{bmatrix}$$

- (d) Consider the binary tree  $T$  in the following figure. (i) Find the depth  $d$  of  $T$ . (ii) Traverse  $T$  using the post-order algorithm.



- (e) Suppose a graph  $G$  contains two distinct paths from a vertex  $u$  to a vertex  $v$ . Show that  $G$  has a cycle.  
 (f) Consider the binary tree  $T$  in the following figure. Describe the tree  $T$  after (i) the node  $M$  and (ii) the node  $D$  are deleted.



5. Answer any *three* of the following:

(15 M)

- Draw the Hasse diagram for divisibility on the set  $\{1, 2, 3, 5, 7, 11, 13\}$ .
  - How many solutions does the equation  $x+y+z=11$  have, where  $x, y$  and  $z$  are non-negative integers with  $x \leq 3, y \leq 4$  and  $z \leq 6$ ?
  - Draw all possible non similar binary trees  $T$  with four external nodes.
  - Show that  $a_n = n \cdot 2^n$  is a solution of the non-homogeneous linear recurrence relation  $a_n = 2a_{n-1} + 2^n$ .
  - What is the language generated by phase structure grammar  $G$ ?
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