FY-CS - 48

Q.P. Code :05602

[Time: 2 $^1/_2$ Hours] **[Marks:75]**

Please check whether you have got the right question paper.

N.B: 1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 Answer following questions. **15**

a) Choose the best choice for the following questions: **05**

i) Let f be a function that is continuous on $[a, b]$ and differentiable on $[a, b]$. If $f''(x) = 0 \forall x \in (a, b)$, then f is .... on $[a, b]$.

p) increasing
q) decreasing
r) constant
s) None of these

ii) If a function f is concave down on (a, b), which of the following is true on (a, b).

p) $f' > 0$
q) $f' < 0$
r) $f' = 0$
s) None of these

iii) If f and g are integrable functions on $[a, b]$ and $f(x) \geq g(x)$ for all $x \in [a, b]$, then

p) $\int_a^b f(x) \geq \int_a^b g(x)$

q) $\int_a^b f(x) \leq \int_a^b g(x)$

r) Either (p) or (q)
s) Neither (p) nor (q)

iv) A rule that assigns a unique real number $f(x, y, z)$ to each point (x, y, z) in some set D in the xyz-surface is called

p) a function of one variable
q) a function of two variables
r) a function of three variables
s) None of these.

v) which of the following is true about the function $f(x, y) = \frac{x^3y^2}{1-xy}$?

p) continuous everywhere
q) Continuous except where $1 - xy = 0$
r) Either (p) or (q)
s) Neither (p) nor (q)

(Turn Over)

b) Fill in the blanks for the following question: **05**

i) A function f has a relative minimum at x_0 if there is an open interval containing x_0 on which $f(x)$ is ……………$f\ (x_0)$ for every x in the interval.

ii) If $f''\ (a)$ exists and f has an inflection point at $x = a$, then $f''\ (a)$ is ……….

iii) If a function f is smooth on $[a, b]$, then the length of the curve $y = f\ (x)$ over $[a, b]$ is ……….

iv) A solution of a differential equation $\frac{dy}{dx} - y = e^{2x}$ is given by ……….

v) If $f\ (x, y, z) = \sqrt{1 - x^2 - y^2 - z^2}$, the value of $f\left(1, \frac{1}{2}, \frac{-1}{2}\right)$ is given by ………….

c) State true of false for the following questions: **05**

i) If a function f is continuous on $[a, b]$, then f has an absolute maximum on $[a, b]$.

ii) Newtons Method is a process to find exact solutions to $f(x) = 0$.

iii) The equation $\left(\frac{dy}{dx}\right)^2 = \frac{dy}{dx} + 2y$ is an example of a second order differential equation.

iv) If $g(x)$ is continuous at x_0 and $h(y)$ is continuous at y_0, then $f(x, y) = g(x)h(y)$ is continuous at (x_0, y_0)

v) A function f of two variables is said to have a relative minimum at a point (x_0, y_0) if there is a disk centered at (x_0, y_0) such that $f(x_0, y_0) \geq f(x, y)$ for all points (x, y) that lie inside the disk.

Q.2 Answer any THREE of the following questions: **15**

a) Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.

b) Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

c) Locate the critical points of $f(x) = 4x^4 - 16x^2 + 17$.

d) Find the absolute maximum and minimum values of $f(x) = 8x - x^2$ in [0,6].

e) A liquid form of antibiotic manufactured by a pharmaceutical firm is sold in bulk at a price of Rs 200 per unit. If the total production cost (in Rs) for x units is $C(x) = 500{,}000 + 80x + 0.003x^2$ and if the production capacity of the firm is at most 30,000 units in a specified time, how many units of antibiotic must be manufactured and sold in that time to maximize the profit?

f) The equation $x^3 + x - 1 = 0$ has one real solution. Approximate it by Newtons Method.

Q.3 Answer any THREE of the following questions: **15**

a) Find the area under the curve $y = x^4$ over the interval$[-1,1]$.

b) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

c) Find the approximate value of $\int_1^2 \frac{1}{x^2} dx$ using Simpson's rule with n=10.

d) Solve differential equation $\frac{dy}{dx} = -xy$

e) Use Euler's Method with a step size of 0.2 to find approximate solution of the initial-value problem $\frac{dy}{dx} = y - x, y(x) = 2$ over $0 \leq x \leq 1$.

f) Solve the differential equation $\frac{dy}{dx} + y = \frac{1}{1+e^x}$ by the method of integrating factors.

(Turn Over)

Q.4 Answer any THREE of the following questions: 15

a) Let $f(x,y) = -\frac{xy}{x^2+y^2}$. Find limit of $f(x,y) as\ (x,y) \to (0,0)$ i) Along y-axis and ii) along the line y = -x.

b) Evaluate $lim_{(x,y)\to(0,0)} y.\log(x^2+y^2)$, by converting to polar coordinates.

c) Find $f_x(1,3)$ and $f_y(1,3)$ for the function $f(x,y) = 2x^3y^2 + 2y + 4x$.

d) Find the directional derivative of $f(x,y,z) = x^2y - yz^3 + z$ at the point (1, 2, 0) in the direction of the vector $a = 2i + j - 2k$.

e) Find an equation of the tangent plane to the surface $z = x^2y$ at the point (2, 1, 4). Also find the parametric equation of the line that is normal to the surface at the point (2, 1, 4).

f) Find all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

Q.5 Answer any THREE of the following questions: 15

a) Let $f(x) = ax^2 + bx + c$, where $a > 0$. Prove that $f(x) \geq 0$ for all x if and only if $b^2 - 4ac \leq 0$.

b) Show that $y = xe^{-x}$ satisfies the equation $xy' = (1-x)y$.

c) Find the area of the region under the curve $y = x - x^2 + 1$ and above the x-axis.

d) Solve differential equation $x\frac{dy}{dx} - y = x$.

e) Determine whether the following limit exists. If so, find its value. $lim_{(x,y)\to(0.0)} \frac{x^4-y^4}{x^2+y^2}$.