Q.P. Code: 31173

(2 ½ Hours)   [Total Marks: 75]

N.B.   1) All questions are compulsory.
        2) Figures to the right indicate marks.
        3) Illustrations, in-depth answers and diagrams will be appreciated.
        4) Mixing of sub-questions is not allowed.

Q. 1   Attempt All (Each of 5Marks)   **(15M)**

(a)   **Select correct answer from the following:**

1) In which of the following method, we approximate the curve of solution by the tangent in each interval.
a) Simpson's Method
b) Euler's method
c) Newton's method
d) None of the above

2) $\int 1/(9x^2 + 25) \ dx =$
a) $(3/5) \tan^{-1}(3x/5) + c$
b) $(1/9) \tan^{-1}(3x/5) + c$
c) $(3/5) \tan^{-1}(5x/3) + c$
d) $(1/15) \tan^{-1}(3x/5) + c$

3) A function is said to be invertible if and only if it is_____
a)Bijective   b) injective   c) Inflexion   d) Surjective

4) $\lim\limits_{x \to \infty} 7/2x =$
a)1   b)infinite   c) zero   d) None

5) If $f(x, y) = x^3y^3 + y^3 + 1$ then $f_x(x, y)$ is
a)$3x^2$ b) $3xy$ c) y^3x d) None

(b)   **Fill in the blanks:**
(*continuous*, ∞, (4i+5j)/41, (4i+5j)/31, $-\infty$, e^x, derivative, $x - 3 \log|x + 3| + c$)
   1. $\lim\limits_{x \to \infty} (5 - 2x) =$ _____.
   2.   The derivative of e^x is _____.
   3.   Unit vector of 4i+5j is_____.
   4. $\int x/(x +3) \ dx =$ _____
   5.   The rate of change of one variable with respect to another is called _____.

(c)   **Answer the following in one line**
1. Define Tangent Plane
2. Define Critical Point
3. Define the term Definite Integral
4. Evaluate $\int_{\pi/3}^{2\pi} \sin x \ dx$
5. Linearization of a function

Q. 2   Attempt the following (Any THREE)   (15M)
(a)   Show that $\lim\limits_{x \to 1} 2x^2 + 3x - 4 = 1$
(b)   Discuss the continuity of the function $f(x) = \sqrt{4 - x^2}$
(c)   Show that the function $f(x) = x^3 - 9x^2 + 30x + 7$ is always increasing.
(d)   Find the relative extrema of $f(x) = 4xy - x^4 - y^4$ using both first and second derivative test.
(e)   Using Newton's method find the approximate root for the equation
      $f(x) = x - \cos x$
(f)   Divide 100 into two parts such that sum of their square is minimum.

Q. 3   Attempt the following (Any THREE)   (15M)
(a)   Evaluate $\int \sin^{-1}\sqrt{x} \ dx$

(b)   Evaluate $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \dfrac{1}{(1+\cot x)} \ dx$

(c)   Estimate $\int_0^4 x^2 \ dx$ using simpson's rule and n = 4.

(d)   Solve the differential equation
      $\sec^2 x \tan y \ dx + \sec^2 y \tan x \ dy = 0$

(e)   Solve $dy/dx = 1 - y$; $y(0) = 0$, find $y(0.1)$ and $y(0.3)$ using Euler's method. Taking h = 0.1.
(f)   Solve the differential equation
      $(x + 1)\dfrac{dy}{dx} - y = e^x (x + 1)^2$

Q. 4   Attempt the following (Any THREE)   (15)
(a)   Show that $f(x, y) = 2x^2 + 3xy$ is continuous at (2, 3)
(b)   Find the second order derivatives of $f(x.y) = x^2 y^3 + x^4 y$
(c)   If $z = x^2 y$, $x = t^2$ and $y = t^3$ Use chain rule to find $\frac{dz}{dt}$.
(d)   Find the directional derivative of $f(x, y) = x^3 + 2xy^2$ at the point (-2, -3) in the direction of the vector $a = i + j$
(e)   Find the gradient vector of $f(x, y)$ if $f(x, y) = 10 - 8x^2 - 2y^2$. Evaluate it at (2, 3)
(f)   Find the equation for the tangent plane and parametric equations for normal line to the surface $z = x^2 y$ at the point (2, 1, 4)

Q. 5   **Attempt the following (Any THREE)**   (15)

(a)   Locate all relative extrema and saddle points of
        $f(x, y)= 3x^2 - 2xy + y^2 - 8y$

(b)   Solve the differential equation
        $$\frac{dy}{dx} = (4x + y + 1)^2$$

(c)   Draw the graph of $y= 4 - 3x^2 + x^3$ and find the intervals on which the function y is
        increasing and decreasing(draw the graph on the answer sheet itself)

(d)   Find the asymptotes of the function $y= \frac{x}{(x+1)(x+2)^2}$

(e)   Solve the differential equation
        $dy/dx = (4x + y + 1)^2$

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