(2½ hours)

**Total Marks: 75**

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

**1. Attempt *any three* of the following:** **15**

a. Find the adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

b. Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} -17 & 18 & -6 \\ -18 & 19 & -6 \\ -9 & 9 & 2 \end{bmatrix}$$

c. Discuss the consistency of the following systems of equations and solve them whenever possible.
$X_1 + 2X_2 + 2X_3 = 1$
$2X_1 + 2X_2 + 3X_3 = 3$
$X_1 - X_2 + 3X_3 = 5$

d. Express in a + ib form $\cot(x + iy)$.

e. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$.

f. Prove that $(1 + \cos x + i \sin x)^n = 2^n \cos^n x/2 \,(\cos nx/2 + i \sin nx/2)$

**2. Attempt *any three* of the following:** **15**

a. Solve the Differential Equation $(1 - 2xy - x^3)\,dy - (1 + y^2 + 3x^2y)\,dx = 0$

b. Solve the Differential Equation $x^2 dy/dx = 3x^2 - 2xy + 1$

c. Solve the following Equation $\sec x \, dy/dx = y + \sin x$

d. Solve the following Equation $p^2x(x-2) + p(2y - 2xy - x + 2) + y^2 + y = 0$

e. Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$.

f. Find the General Solution of the equation $(D^3 + 3D)y = \cos x$

**3. Attempt *any three* of the following:** **15**

a. Evaluate $\int_0^\infty e^{-3t} t \cos 2t \, dt$

b. Find the inverse Laplace transform for the function

$$F(s) = \frac{5s+3}{(s-1)(s^2+2s+5)}$$

**[TURN OVER]**

c. Find Laplace transformation of the function

$$f(t) = t(2\sin 3t + e^{2t})$$

d. Obtain the Laplace transform of each of the given function
$F(t) = e^{-2t}\cos 4t + e^{3t}\sin 6t$

e. Find Inverse Laplace Transformation by convolution theorem for

$$F(s) = \frac{s^2}{(s^2 + a^2)^2}$$

f. Using Laplace transform method solve the following differential equations with the given condition.
$(D^2 + 3d + 2)y = 4t + e^{3t}$ if $y = 1$, $Dy = -1$ at $t = 0$.

**4. Attempt *any three* of the following:** **15**

a. $Evaluate \int_0^2 \int_x^{4-x} \int_0^1 e^{2x+2y}\,dxdy.$

b. $$Evaluate \int_0^4 \int_0^{\sqrt{4x-x^2}} \frac{ydxdy}{(x^2+y^2)^{1/2}}$$

c. $Evaluate \int_0^1 \int_{y*y}^1 \int_0^{1-x} x\,dxdydz.$

d. $Evaluate \int_0^a \int_0^{(a2-x2)1/2} \int_0^{(a2-x2-y2)1/2} (xyz)\,dxdydz$ .

e. Change the order of integration and evaluate $\int_{-1}^{2} \int_{x^2}^{x+2} dxdy$

f. Change to polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}dxdy$ .

**5. Attempt *any three* of the following:** **15**

a. Evaluate $\int_0^\pi \sin^2 x\,(1+\cos x)^4\,dx.$

b. Evaluate $\int_0^\infty \frac{x^2 dx}{(1+x^6)^{7/2}}$.

**[TURN OVER]**

c. Find Laplace transformation of the function

$$f(t) = t(2\sin 3t + e^{2t})$$

d. Obtain the Laplace transform of each of the given function
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**5. Attempt *any three* of the following:** **15**

a. Evaluate $\int_0^\pi sin^2x\,(1+\cos x)^4\, dx.$

b. Evaluate $\int_0^\infty \frac{x^2 dx}{(1+x^6)^{7/2}}$.

c. Evaluate $\int_0^\infty e^{-\alpha x} \sin x / x \;\; dx$

d. Evaluate $\int_0^{\pi/2} \frac{\log(1+a\, sin^2x)dx}{sin^2x}$.

e.