S4PT | 85

Q. P. Code: 20945

(Time: 2½ hours)

Total Marks: 75

N. B.: (1) **All** questions are **compulsory**.
(2) Make **suitable assumptions** wherever necessary and **state the assumptions** made.
(3) Answers to the **same question** must be **written together**.
(4) Numbers to the **right** indicate **marks**.
(5) Draw **neat labeled diagrams** wherever **necessary**.
(6) Use of **Non-programmable** calculators is **allowed**.

1. Attempt *any three* of the following: **15**

a. Find the Adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} 2 & -1 & 3 \\ 4 & 6 & -2 \\ 5 & 1 & 8 \end{bmatrix}$$

b. Find the Characteristic values and characteristic vectors of the given matrix.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

c. Verify Caley-Hamilton theorem for the given matrix, also find inverse if exists.

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

d. Expand $(1 + \cos x + i\sin x)^n$

e. Evaluate $(1 + i\sqrt{3})^{16} / (\sqrt{3} - i)^{17}$

f. Express $\sec(x + iy)$ in $a + ib$ form

2. Attempt *any three* of the following: **15**

a. Solve the Differential Equation $(x - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$

b. Solve the Differential Equation $dy/dx + x^2y = x^5$

c. Solve the following Equation $x^2p^2 - 2xpy + (2y^2 - x^2) = 0$

d. Solve the following Equation $p(p + y) = x(x + y)$

e. Find the Complementary and Particular Solution of the equation $(D^3 + D^2 + D + 1)y = \sin 2x$

f. Find the General Solution of the equation $(D^2 + 4)y = \sin 3x + e^x + x^2$

3. Attempt *any three* of the following: **15**

a. Evaluate $\int_0^\infty e^{-2t} \sin^2 t\, dt$

b. Find the inverse Laplace transform for the function

$$F(s) = \frac{21 - s^2}{s(s^2 + 4s + 13)}$$

c. Find Laplace transformation of the function

$$f(t) = te^{2t} \cos 3t$$

[TURN OVER]

d. Obtain the Inverse Laplace transform of each of the given function

$$\frac{(s+1)}{s^3(s-3)^2}$$

e. Find Inverse Laplace Transformation by convolution theorem for

$$F(s) = \frac{s}{(s^2+1)(s^2+4)}$$

f. By using fundamental definition, find laplace transform of f(t)

F(t) = t, $0 < t < 4$

= 5, $t > 4$

4. Attempt *any three* of the following: **15**

a.

$$Evaluate \int_0^1 \int_0^2 e^{x+y} dxdy$$

b.

$$Evaluate \int_0^3 \int_0^{\sqrt{4-y}} \frac{dxdy}{(1+x^2+y^2)}$$

c.

$$Evaluate \int_0^{log2} \int_0^x \int_0^{x+logy} e^{x+y+z}\, dxdydz$$

d.

$$Evaluate \int_0^1 \int_0^{1-x} \int_0^{x+y} e^z\, dxdydz$$

e. Change the order of integration and evaluate $\int_0^2 \int_0^{x^2/4} xydxdy$

f. Solve $\iint r^3\, drd\theta$ over the area included between the circles r = 2sinθ and r = 4 sinθ

5. Attempt *any three* of the following: **15**

a.

Evaluate $\int_0^{\pi/2} sin^6x\, cos^7x\, dx$

b. Evaluate i) erfc(-x) + erfc(x)

ii) erfc(x) + erf(x)

c.

Evaluate $\int_0^{2a} x(2ax - x^2)^{1/2} dx$

[TURN OVER]

d. Evaluate $\int_0^{\pi/2} sin^5\, 2x dx$

e. Evaluate $\int_0^1 \frac{x^7}{(1-x^4)^{1/2}} dx$

f. Evaluate $\int_0^1 \frac{(x^a - x^b)}{logx} dx$
