Duration: 2:30 Hrs. G215OAM19 Maximum Marks: 75

Note : 1. All questions are compulsory.
2. Figures to right indicate full marks.
3. Use of Non- Programable calculator is allowed.

Q.1. Attempt any three of the following. **(15)**

a. Find the adjoint of the given matrix and hence find the inverse of matrix, if exists

$$\begin{matrix} 1 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{matrix}$$

b. Find the characteristic equation and characteristic values of the given matrix

$$\begin{matrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{matrix}$$

c. Verify Cayley-Hamilton theorem for matrix A

$$A = \begin{matrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{matrix}$$

d. Solve the equation $x^7 + x^4 + x^3 + 1 = 0$

e. Express in the form $a + ib, \frac{2-\sqrt{3}i}{1+i}$

f. If $2\cos\theta = x + \frac{1}{x}, Prove\ that\ 2\cos r\theta = x^r + \frac{1}{x^r}$

Q.2. Attempt any three of the following. **(15)**

a. Solve: $(x^2 - yx^2)\,dy + (y^2 + xy^2)\,dx = 0$

b. Solve: $sin\,2x\frac{dy}{dx} = y + tan\,x$

c. Solve: $x^2p^2 + xyp - 6y^2 = 0$

d. Solve: $(D^4 - 3D^2 - 4)y = 5\,sin\,2x - e^{-2x}$

e. Solve: $(D^2 - D + 1)y = x^3 - 3x^2 + 1$

f. Solve: $y = 2px + x^2 + p^4, where\ p = \frac{dy}{dx}$

Q.3. Attempt any three of the following. **(15)**

a. Find Laplace transform of $f(t) = te^{3t}\cos 2t$

b. Find Laplace inverse transform for the function $f(s) = \frac{5s+3}{(s+1)(s^2+2s+5)}$

c. Find Laplace transform of $\left(\frac{cos\,at - cos\,bt}{t}\right)$

d. Find the Laplace transform of $\int_0^t \frac{sin\,t}{t}\,dt$

e. Find Inverse Laplace inverse transform for the function $f(s) = \left(log\left(\frac{s^2-1}{s}\right)\right)$

f. Solve by Laplace transform

$$\frac{d^2y}{dt^2} + 4y = 8\,sin\,t, given\ y(0) = 1, y^1(0) = 2$$

Q.4. Attempt any three of the following. (15)

a. Change the order of integration and Evaluate $\int_0^a \int_{\frac{x}{a}}^{\sqrt{\frac{x}{a}}} (x^2 + y^2)\, dx \cdot dy$

b. Evaluate: $\int_1^e \int_1^{\log y} \int_1^{e^x} \log z\, dx \cdot dy \cdot dz$

c. Evaluate $\int\int x^3\, dx\, dy$, over the circle $x^2 + y^2 = a^2$

d. Evaluate: $\int\int\int (x + y + z)\, dx\, dy\, dz$, over the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$

e. Evaluate: $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} dx\, dy$ by changing to polar coordinates

f. Evaluate: $\int_0^2 \int_0^x \int_0^{2x+2y} e^{x+y+z}\, dxdy\, dz$

Q.5. Attempt any three of the following (15)

a. Evaluate: $\int_0^\infty \frac{x^a}{a^x} dx$

b. Find $\int_1^\infty \frac{(x-1)^7}{x^{12}} dx$

c. Evaluate: $\int_2^5 \sqrt{(x-2)^7 (5-x)^9}\, dx$

d. Prove that $erf(-x) + erfc(x) = 2$

e. Prove that $\beta(m, n) = \beta(m, n+1) + \beta(m+1, n)$

f. Define error function and prove that error function is an odd function.