

DURATION: 2 ½ HOURS

G520100AM

MARKS: 75

N. B.: (1) All questions are compulsory.

(2) Make suitable assumptions wherever necessary and state the assumptions made.

(3) Answers to the same question must be written together.

(4) Numbers to the right indicate marks.

(5) Draw neat labeled diagrams wherever necessary.

(6) Use of Non-programmable calculators is allowed.

Q1. Attempt any three of the following:

(15M)

a. Express in $a + ib$ form $\cot(x + iy)$.

b. express in polar form $-1 + \sqrt{3}i$

c. For different values of k , discuss the following equations:

$$x + 2y - z = 0;$$

$$3x + (k + 7)y - 3z = 0;$$

$$2x + 4y + (k - 3)z = 0$$

d. Express in $a + ib$ form $\cot(x + iy)$.

e. Find the adjoint of the given matrix and hence find Inverse if exist

$$\begin{bmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{bmatrix}$$

f. Prove that $(1 + \cos x + i \sin x)^n = 2^n \cos^n \frac{x}{2} (\cos \frac{nx}{2} + i \sin \frac{nx}{2})$

Q2. Attempt any three of the following:

(15M)

a. Solve the Differential Equation $(1 - 2xy - x^3) dy - (1 + y^2 + 3x^2 y) dx = 0$

b. Find the General Solution of the equation $(D^3 + 3D)y = \cos x$

c. Change to polar coordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2 + y^2)} dx dy$

d. find the area of the circle under the region $x^2 + y^2 = 49$

e. solve : $y = xp + 1/p$

f. solve : $(D^2 + 6D + 9)Y = 5^x - \log 2$

Q3. Attempt any three of the following

(15M)

- a. Find the Laplace Transformation of $f(t) = t^3 e^{2t}$
- b. Find Laplace transformation of the function $f(t) = t(2\sin 3t + e^{2t})$
- c. Find inverse Laplace transform $(\tan^{-1}s)$
- d. Evaluate $F(t) = \int_0^t e^{-3t} \cos 2t dt$
- e. Find Laplace transformation of the function $f(t) = t(2\sin 3t + e^{2t})$
- f. Find Inverse Laplace Transformation by convolution theorem for $f(s) = s^2 (s^2 + a^2)^{-2}$

Q4. Attempt any three of the following

(15M)

- a. Explain the Legendre's differential equation
- b. Define error function. Evaluate $\operatorname{erf}(\sqrt{x})$
- c. Solve the following equation
 $X+2Y+3Z=0, 2X+3Y+Z=0, 4X+5Y+4Z=0$
- d. Find the area bounded by region if $Y = 2X$ and $y^2 = 16ax$
- e. What is Laplace transform and state its advantages
- f. Find the Laplace transform of $t \cdot \sin 6t$

Q5. Attempt any three of the following

(15M)

- a. Find the area of ellipse $x^2/16 + y^2/49 = 1$
- b. Find the complex square root of $3-2i$
- c. Derive the formula of Laplace transform of $\sin at$
- d. Prove that error function is an odd function
- e. Find the Laplace transform of $(t \cos 3t + 1 - \sinh 6t)$
- f. Explain types of differential equation and write advantages of D.E