S.Y. CS Sem-III March-2023 ATKT only one paper OLA available

Time: 2½ Hours | H72110OLA | [Total Marks:75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt the following questions **(15M)**

(a) Choose the best choice for the following questions **(5M)**

(i) The absolute value of 3 + 4i is:
a) 4 b) 5 c) 6 d) zero

(ii) In GF (2) field, 1 + 1 is equal to
a) 1 b) 0 c) both a) and b) d) none of these

(iii) How to declare the complex number in Python?
a) (3, 4) b) Complex (3, 4) c) Complex (3, 4i) d) None of these

(iv) If a matrix is R × C and a vector is a C vector then the product is called
a) Matrix-Matrix b) Vector-Matrix c) Vector-Vector d) Matrix-Vector

(v) Suppose t = (1, 2, 4, 3), which of the following is incorrect?
a) print (t [3]) b) t [3] = 45 c) print(max(t)) d) print(len(t))

(b) Fill in the blanks for the following questions **(5M)**

i) Any complex number multiplying by i, rotate it by ___________
ii) Set of all linear combinations of vectors is called __________
iii) A rectangular array of m rows and n columns is called a ___________
iv) Norm of Vector (1, 2, 3) is ____________
v) Every Subset of a linearly independent set is linearly ____________

(c) Define. **(5M)**

i) Dot product.
ii) Gatois field.
iii) Eigen Value.
iv) Orthogonal Complement.
v) Dimension.

Q. 2 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Find the square root of complex number 8 – 6i

(b) Determine whether v_1= (2, 2, 2), v_2= (0, 0, 3) and v_3= (0, 1, 1) span vector space R 3.

(c) Let W_1 and W_2 are two subspaces of V then prove that $W_1 \cap W_2$ is also a subspace of V where V is a vector space on IR.

(d) Write a python Program for rotating a complex number

Z = 2+3i by 180?

(e) Express [(3 + 2i) / (2 + i) (1 – 3i)] in the form x + iy

(f) Check whether the set of all pairs of real numbers of the form (1, x) with operation (1, y) + (1, y′) = (1, y + y′) and k (1, y) = (1, ky) is a vector space.

Q. 3 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Show that vector {(1, 2, 1), (2, 1, 0), (1, –1, 2)} of R3 form a basis of R3.

(b) Find the co-ordinate representation of vector v = (0, 0, 0,1) in terms of the vectors [1,1,0, 1], [0,1,0, 1] and [1,1,0, 0] in GF (2).

(c) Write a python program to enter a matrix and check if it is invertible. if invertible exists then find inverse.

(c) Let f: U→ V is a linear transformation then show that kerf = {0} if f is injective.

(e) Consider Subspace $U1$ $\{(x, y, w, z) : x - y = 0\}$ and $U2$ $\{(x, y, w, z) : x = w, y = z\}$ Find a basis and dimension of

i) $U1$ ii) $U2$ iii) $U1 \cap U2$.

(f) Find the dimension of the vector space spanned by the vectors (1, 1, –2, 0, –1), (1, 2, 0, –4, 1), (0, 1, 3, –3, 2), (2, 3, 0, –2, 0) and also find the basis.

Q. 4 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Write a python program to find orthogonal projection u on v.

(b) write a program in python to final gcd (240,24)

(c) Let a = (3,0), b = (2,1) find vector in span $\{a\}$ that is closet to b is $b \,||\, a$ and distance $||b \perp a||$.

(d) Find inner product, angle, orthogonality for P = –5 + 2 x – x 2, q = 2 + 3x2

(e) Find the orthonormal basis for subspace IR4 whose generators are $v1$ = (1, 1, 1, 1), $v2$ = (1, 2, 4, 5), $v3$ = (1, −3, −4, −2) Using Gram Schmidt orthogonali sation Method.

(f) Verify Pythagorean Theorem for u = (1, 0, 2, −4) and v = (0, 3, 4, 2)

Q. 5 Attempt the following (Any THREE) (Each of 5Marks) (15M)

(a) Express the following as a linear combination of v1= (−2, 1, 3), v2= (3, 1, −1) and v3= (−1, −2, 1) with w= (6, −2, 5)

(b) Fill the table:

Vector space	Basis	Dimensions
{0}		
IR^2	{(1,0), (0,1)}	
P_2 (X)		3
M_2(IR)		4
IR	{1}	

(c) Express the following as a linear combination of v_1= (−2, 1, 3), v2= (3, 1, −1) and v3= (−1, −2, 1) with w= (6, −2, 5)

(d) Let S be a subset of vector space V. Prove that S ⊥ is a subspace of V.

(e) Construct an orthonormal basis of R^2 by Gram Schmitt Process S = {(3, 1), (4, 2)}

(f) Let T: $|R^3 \rightarrow |R^2$ be a linear map defined by f (x, y, z) = (x+2y-z, x+y-2z) Verify Rank T + Nullity T = 3.