21/3/24 ATKT

24 CS. Linear Algebra (03)

DURATION: - 2½ hrs | 832081123 | MARKS:- 75

Note: - (1) All questions are compulsory.
(2) All questions carry equal marks.
(3) Figures to the right indicates full marks.

**Q.1** **Attempt any 'Four' of the following:** **(20)**

1. Express the following in the standard form of complex number $(x + iy)$ CO1-(U)
   a) $\frac{2-\sqrt{3}i}{1+i}$ b) $\frac{3+2i}{2-3i}$
2. Express the following in polar form and find their arguments. CO1-(U)
   a) $\sqrt{3}+i$ b) $\frac{1}{2}+i\frac{\sqrt{3}}{2}$
3. For $u = [0,4]$ and $v = [-1,3]$, Find vector $v-u, u+v, u-v, 3v-2u$ CO1-(A)
4. For each of the following pair of vectors over R,evaluate the expression u.v: CO1-(A)
   a) $u = [1,2,3]$, $v = [3,2,1]$
   b) $u\ [6,2,-1]$, $v = [5,-8,2]$
5. Find the cross product of the given two vector CO1-(U)
   $\vec{a} = 5\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$ *and* $\vec{b} = \hat{\imath} + \hat{\jmath} + \hat{k}$
6. Solve a triangular system of linear equation where, CO1-(U)
   $x + y + z = 9\,; 2x - y + z = 5$ *and* $4x + y - z = 7$

**Q.2** **Attempt any 'Four' of the following** **(20)**

1. Compute the following matrix-vector products CO2-(U)
   a) $\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \times [0.5,0.5]$ b) $\begin{bmatrix}1 & 2 & 3\\ 1 & 2 & 4\end{bmatrix} \times [7,0,1]$
2. Compute the following CO2-(U)
   a) $\begin{bmatrix}1 & 2 & 3 & 4\\ 1 & 1 & 3 & 1\end{bmatrix} \times \begin{bmatrix}1\\ 2\\ 3\end{bmatrix}$ b) $[2\ 4\ 1] \times \begin{bmatrix}1 & 2 & 0\\ 5 & 1 & 1\\ 2 & 3 & 0\end{bmatrix}$
3. Find a basis for the subspace spanned by 5 vectors. CO2-(U)
   $V_1(1,2,2,-1), V_2(1,3,1,1), V_3(1,5,-1,5), V_4(1,1,4,-1)$ *and* $V_5(2,7,0,2)$
4. Check whether given vector are linearly dependent or not CO2-(U)
   $\{(1,2,0), (2,4,1), (0,0,-1)\}$
5. Show that the following set of vector over R are linearly dependent. CO2-(A)
   $(2,2,1), (-4,6,5), (1,0,0)$
6. Write a Python Program to enter a matrix and check if it is invertible. If invertible exists, then find inverse. CO2(A)

**Q.3** **Attempt any 'Four' from the following** **(20)**

1. Find the eign values of $A = \begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -5 & -2\end{bmatrix}$ CO3 (U)
2. Calculate the eign vectors for the A. CO3 (U)
3. Check whether the matrix A is diagonalizable then find the transforming matrix P CO3 (U)
4. Solve the following system by Gaussion elimination method CO3 (A)
   $x + y + z = 9\,;\ 2x + 5y + 7z = 52\,;\ 2x + y - z = 0$

5. Write a python program to convert a 2*2 matrix to row echelon form. CO3 (A)

6. For the following matrix- vector equations check whether the solution exist or not? If exists then CO3 (A)

Solve $\begin{bmatrix} 1 & 3 & -2 & 1 & 0 \\ 0 & 0 & 2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} * [x_1, x_2, x_3, x_4, x_5] = [5,3,2,1]$

**Q.4 Attempt any 'Five' of the following :** **(15)**

1. Find the minimal polynomial of $A = \begin{bmatrix} 3 & -1 & 0 \\ 0 & -2 & 0 \\ 1 & -1 & 2 \end{bmatrix}$ CO3 (A)

2. Write a short note on Internet worm CO3 (R)

3. Calculate the absolute value of $3 + 4i$ CO3 (U)

4. Define the term inner product space CO3 (R)

5. Express $\frac{3+2i}{1-2i}$ in terms of $x + iy$ CO3 (U)

6. Let $A = \begin{bmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 4 \\ 2 & 2 \\ 1 & 0 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix}$ $D = [2\ 4\ 3\ 1]$ CO3 (A)

Compute a) $A + B$, b) $3A$ C) $B + 2D$ if they exists.

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