Time: 2½ Hours **H72110OLA** **[Total Marks:75]**

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 Attempt the following questions **(15M)**

(a) Choose the best choice for the following questions **(5M)**

i) The absolute value of 3 + 4i is:
a) 4 b) 5 c) 6 d) zero

ii) In GF (2) field, 1 + 1 is equal to
a) 1 b) 0 c) both (a) and (b) d) none of these

iii) How to declare the complex number in Python?
(a) (3, 4) b) Complex (3, 4) c) Complex (3, 4i)
d) None of these

iv) If a matrix is R × C and a vector is a C vector then the product is called
(a)Matrix-Matrix (b) Vector-Matrix c) Vector-Vector
(d) Matrix-Vector

v) Suppose t = (1, 2, 4, 3), which of the following is incorrect?
a) print (t [3]) b) t [3] = 45 c) print(max(t)) d) print(len(t))

(b) Fill in the blanks. **(5M)**

(Spare, Unique, Unit, √45, Inner product)

i) A vector whose norm is one is called _____ vector.
ii) A vector space together with inner product is called _____ space.
iii) If most of the element of a matrix have zero value is called _____ matrix.
iv) The absolute value of 3+6i = _____.
v) Inverse of a matrix is _____.

(C) Define. **(5M)**

i) Dot product.

ii) Gatois field.

iii) Eigen Value.

iv) Orthogonal Complement.

v) Dimension.

Q. 2 Attempt the following (Any THREE) **(15M)**

(a) Solve the following system by backward substitution method

$x1 - 3x2 - 2x3 = 7$

$2x2 + 4x3 = 4$

$10x3 = 20$

(b) Write a python Program for rotating a complex number

Z = 2+3i by 180?

(c) Write a Python program to rotate a complex no by 90^0, 180^0 and 270^0

(d) Express [(3 + 2i)/ (2 + i) (1 – 3i)] in the form x + iy

(e) Which of the following is a set of generators of $IR3$

i) {(4, 0,0), (0,0,2)}

ii) {(1, 0,0), (0,1,0), (0,0,1)}

(f) Let $W1$ and $W2$ are two subspaces of V then prove that $W1 \cap W2$ is also a subspace of V where V is a vector space on IR.

Q. 3 Attempt the following (Any THREE) (Each of 5Marks) **(15M)**

(a) Let f: U→ V is a linear transformation then show that kerf = {0} if f is injective.

(b) Find the co-ordinate representation of vector v = (0, 0, 0,1) in terms of the vectors [1,1,0, 1], [0,1,0, 1] and [1,1,0, 0] in GF (2).

(c) Check whether the set of functions are Linearly independent? 2 – x + 4x2 , 3 + 6x + 2x2 , 2 + 10 x – 4x2 .

(d) If V and W are two subsets of a vector space V such that U is a subset of W then show that $W0$ is a subset of U 0 where $U0$,$W0$ are annihilator of U and W respectively

(e) Find the dimension of the vector space spanned by the vectors (1, 1, –2, 0, –1), (1, 2, 0, –4, 1), (0, 1, 3, –3, 2), (2, 3, 0, –2, 0) and also find the basis.

(f) Write a python program to enter a matrix and check if it is invertible. if invertible exists then find inverse.

Q. 4 Attempt the following (Any THREE) (Each of 5Marks) **(15M)**

(a) Write a python program to find orthogonal projection u on v.

(b) Explain Internet Worm

(c) Let a = (3,0), b = (2,1) find vector in span $\{a\}$ that is closet to b is $b\,||a$ and distance $||b \perp a||$.

(d) Find inner product, angle, orthogonality for P = –5 + 2 x – x 2, q = 2 + 3x2

(e) Let a = (3,0), b = (2,1) find vector in span $\{a\}$ that is closet to b is $b\,||a$ and distance $||b \perp a||$.

(f) Solve the following system by Gaussian elimination method:

$$y - z = 3$$

$$-2x + 4y - z = 1$$

$$-2x + 5y - 4z = -2$$

Q. 5 Attempt the following (Any THREE) (Each of 5Marks) **(15M)**

(a) Write a python program to convert a 2 × 2 matrix to row echelon form

(b) Fill the table:

Vector space	Basis	Dimensions
{0}		
IR^2	{(1,0), (0,1)}	
$P_2(X)$		3
$M_2(IR)$		4
IR	{1}	

(c) Express the following as a linear combination of v_1= (–2, 1, 3), v2= (3, 1, –1) and v3= (–1, –2, 1) with w= (6, –2, 5)

(d) Verify Cauchy's Schwartz's inequality u = (1, 2, –1) and v = (3, 2, -1)

(e) Construct an orthonormal basis of R^2 by Gram Schmitt Process S = {(3, 1), (4, 2)}

(f) Let T: $|R^3 \rightarrow |R^2$ be a linear map defined by f (x, y, z) = (x+2y-z, x+y-2z) Verify Rank T + Nullity T = 3.