Q. P. Code: 34332

(2½ Hours) [Total Marks: 75]

N.B. 1) All questions are compulsory.
2) Figures to the right indicate marks.
3) Illustrations, in-depth answers and diagrams will be appreciated.
4) Mixing of sub-questions is not allowed.

Q. 1 **Attempt the following questions** (15M)
(a) Choose the best choice for the following questions (5M)
  i) The absolute value of 3 + 4i is:
    a) 4   b) 5   c) 6   d) zero

  ii) In GF(2) field, 1 + 1 is equal to
    a) 1   b) 0   c) both a) and b)   d) none of these

  iii) How to declare the complex number in Python?
    a) (3, 4)   b) Complex(3, 4)   c) Complex (3, 4i)   d) None of these

  iv) If a matrix is R × C and a vector is a C vector then the product is called
    a) Matrix-Matrix   b) Vector-Matrix
    c) Vector-Vector   d) Matrix-Vector

  v) Suppose t =(1, 2, 4, 3), which of the following is incorrect?
    a) print(t[3])   b) t[3] = 45
    c) print(max(t))   d) print(len(t))

(b) Fill in the blanks for the following questions (5M)
  i) Any complex number multiplying by i, rotate it by _____
  ii) Set of all linear combinations of vectors is called _____
  iii) A rectangular array of m rows and n columns is called a _____
  iv) Norm of Vector (1, 2, 3) is _____
  v) Every Subset of a linearly independent set is linearly _____

(c) Answer the following questions (5M)
  i) Solve: $1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 1$
  ii) Find dot product of (1, 2), (3, 4)
  iii) Show with example matrix representation in python
  iv) Define the term Basis
  v) Define the term Inner Product Space

Q. 2 **Attempt the following (Any THREE)(Each of 5Marks)** (15M)
(a) Find the square root of complex number −5 + 12i
(b) Show that vectors v_1=(1, 0, 1), v_2=(2, 1, 4) and v_3=(1, 1, 3) do not span vector space.
(c) Write a Python program to rotate a complex no by 90⁰, 180⁰ and 270⁰

(d) Check whether the vectors are linearly dependent
v_1=(1, –2, 1), v_2=(2, 1, –2) and v_3=(7, –4, 1).

(e) Express $[(3 + 2i)/(2 + i)(1 – 3i)]$ in the form $x + iy$

(f) Check whether the set of all pairs of real numbers of the form (1, x) with operation (1, y) + (1, y') = (1, y + y') and k (1, y) = (1, ky) is a vector space.

Q. 3 Attempt the following (Any THREE) (Each of 5Marks)   **(15M)**

(a) Let

$$A = \begin{pmatrix} 2 & 2 \\ 1 & 1 \\ 0 & 6 \end{pmatrix} \qquad B = \begin{pmatrix} 5 & 4 \\ 2 & 2 \\ 1 & 0 \end{pmatrix} \qquad c = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \qquad D = [2 \quad 4 \quad 3 \quad 1]$$

Compute the following if they exists.
a) A +B   b) 3A   c) B + 2D

(b) Find the dimension of the vector space spanned by the vectors (1, 1, –2, 0, –1), (1, 2, 0, –4, 1), (0, 1, 3, –3, 2), (2, 3, 0, –2, 0) and also find the basis.

(c) Check whether the set of functions are Linearly independent?
$2 – x + 4x^2, 3 + 6x + 2x^2, 2 + 10 x – 4x^2$.

(d) Explain Matrix-Vector and Vector-Matrix multiplication with example.

(e) Write a python program to enter a matrix and check if it is invertible. if invertible exists then find inverse.

(f) Show that vector {(1, 2, 1), (2, 1, 0), (1, –1, 2)} of R^3 form a basis of R^3

Q. 4 Attempt the following (Any THREE) (Each of 5Marks)   **(15M)**

(a) If u = (2, 3, –1) and v = (6, –3, –2)
Find a) d(u, v) b) u – v   c) 2u + 3v

(b) Verify Pythagorean Theorem for u = (1, 0, 2, –4) and v = (0, 3, 4, 2)

(c) If x, y, z ≥ 0
Show that $(x^2 + y^2 + z^2)^{1/2} \geq (1/13)(3x + 4y + 12z)$

(d) Find inner product, angle, orthogonality for
$P = –5 + 2 x – x^2, q = 2 + 3x^2$

(e) Find the vector orthogonal to both u = (–6, 4, 2) and v = (3, 1, 5)

(f) Write a python program to find orthogonal projection u on v.

Q. 5 Attempt the following (Any THREE) (Each of 5Marks)   **(15M)**

(a) Express the following as a linear combination of v_1=(–2, 1, 3), v_2=(3, 1, –1) and v_3=(–1, –2, 1) with w= (6, –2, 5)

(b) Write a python program to convert a 2 × 2 matrix to row echelon form

(c) Verify Cauchy's Schwartz's inequality u = (1, 2, −1) and v = (3, 2, -1)

(d) Find eigen Values and eigen vectors of

$$A = \begin{pmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{pmatrix}$$

(e) Construct an orthonormal basis of R^2 by Gram Schmitt Process
S = {(3, 1), (4, 2)}